THE NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

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[Continued from May Number.]

Scholion.

Nevertheless it might be doubted, whether, from whatever point K (assumed indeed in BX before the meeting of this BX with the other AX) erected toward the parts of the straight AX, a perpendicular must meet this (fig. 29) in some point L; provided of course those two, before the aforesaid meeting, are assumed ever more to approach each other mutually [and not to meet at any finite remove].

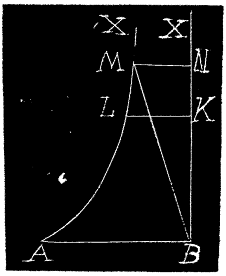

Fig. 29.

But I say it will follow completely thus.

Proof. Let there be assigned in BX any point whatever K. In AX is taken a certain AM equal to the sum of this BK and of twice AB.

Then from the point M is drawn to BX (according to Eu. I. 12) the perpendicular MN. According to the present supposition, MN will be less than AB. Wherefore AM (made equal to the sum of BK and of double AB) will be greater than the sum of BK, AB, and NM. Now it behooves to show this same AM to be less than the sum of BN, AB, and MN, that thence it may follow this BN is greater than the aforesaid BK, and therefore the point K lies between the points B and N.

Join BM. The side AM will be (from Eu. I. 20) less than the two remaining sides together AB and BM. Again the side BM (from the same Eu. I. 20) will be less than the two sides together BN and MN. Therefore the side AM will be by much less than the three sides together AB, BN, and NM. But this was to be shown, in order to deduce that the point K lies between the points B and N. Thence however it follows, that the perpendicular from the point K erected toward the parts of AX must meet this in some point L stationed between the points A and M; else obviously (against Eu. I. 17) it must cut either AB or MN perpendiculars to BX.

Quod etc.

[To be Continued.]